Functions and Limits

1

Definitions:

1. Function:

A function is a rule that assigns to each element x in X a unique element y in Y.

Example: $A = x^2$ (A is a function of x)

domain 4 کی $4^2 = 16$ اور
function x^2 کی range 16 ہے۔

2. Domain:

In a function $f: X \rightarrow Y$ the set X is called the domain of f.

3. Range:

In a function $f: X \rightarrow Y$ the set of corresponding elements y in Y is called the range of f.

4. Independent and dependent Variables:

In $y = f(x)$, the variable x is called independent variable and y is called dependent variable of f.

5. Real valued Function:

If variables used in function are real numbers then function is called real valued function.

6. Algebraic Functions:

Functions which are defined by algebraic expressions.

7. Polynomial Function:

A function of the form $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ where $a_n, a_{n-1}, a_{n-2}, \ldots, a_1, a_0$ are real numbers and exponent are non-negative integers is called polynomial function.

8. Linear Function:

Sgd 2011, Fsd 2009, Sahi 2016, Guj 2016, Bwl 2018

If degree of polynomial function is one then it is called linear function.

Example: $f(x) = 3x + 4$

9. Identity Function:

Sargodha 2011, Rwp 2006

For any set X, a function $I: X \rightarrow X$ of the form $I(x) = x$ is called identity function

Example : $f(x) = x$

10. Constant Function:

A function $C: X \rightarrow Y$ defined by $C(x) = a$ is called constant function.

Example : $c(x) = 2$

Dgk 2017

11. Rational Function:

A function of the form $\frac{P(x)}{Q(x)}$ where P (x) and Q (x) are polynomial functions and $Q(x) \neq 0$ is called rational function:

12. Exponential Function:

Sargodha 2018

A Function in which the variable appears as exponent is called exponential function.

Example: $y = e^x$, $y = 2^x$

13. Logarithmic Function:

Federal 2016

If $x = a^y$ then $y = \log_a x$ $(a>0, a \neq 1)$ is called logarithmic function. If $a = 10$ then $y = \log_{10} x$ is called common log.

If $a = e$ then $y = \log_e x = \ln x$ is called natural log.

14. Explicit Function:

Sargodha 2008, Rwp 2008, Mul 2018

If y is easily expressed in term of independent variable x then y is called explicit function.

Example: $y = x^2 + 2x - 1$

15. Implicit Function:

Sargodha 2008, Rwp 2013, Guj 2018

If y is not easily expressed in term of independent variable x then y is called implicit function.

Example: $x^2 + xy + y^2 = -1$

16. Even Function:

Sargodha 2015, Mtn 2017, Fsd 2017

A function is called an even function if $f(-x) = f(x)$

17. odd Function:

Sargodha 2014, 15, Mtn 2017, Fsd 2017

A function called an odd function if $f(-x) = -f(x)$

18. Continuous Function:

Sargodha 2015, 16, Bahawalpur 2016, Mul 2018

A function f is continuous at c if satisfy three conditions

I. f (c) is defined

II. $\lim_{x \to c} f(x)$ *exists*

III. $\lim_{x \to c} f(x) = f(c)$

1. $\text{Sin hx} = \frac{e^x - e^{-x}}{2}$
2. $\text{Cos hx} = \frac{e^x + e^{-x}}{2}$
3. $\text{Tan hx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
4. Even Function $f(-x) = f(x)$ Sgd 2015
5. Odd Function $f(-x) = -f(x)$ Sgd2014,16
6. Perimeter of Square = $4x$
7. Area of Square = x^2
8. Area of Circle = πr^2
9. Circumference of Circle = $2\pi r$
10. Volume of cube = $V=x^3$
11. $fog(x) = f(g(x))$
12. $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$
13. $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$
14. $\lim_{x \to 0} (1+x)^{1/x} = e$
15. $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a = \ln a$
16. A function f is continuous at c if satisfy three conditions Sgd 2015
    i. $f(c)$ is defined
    ii. $\lim_{x \to c} f(x)$ exists
    iii. $\lim_{x \to c} f(x) = f(c)$
17. Discontinuous if one or more above three conditions are not satisfied.
18. Limit exist if L.H. Limit = R.H. Limit

DIFFERENTIATION

2

Definitions:

1. Average Rate of change: (Sargodha 2011, Fsd 2010)

Let f be a real valued function then (difference quotient) $\frac{f(x_1)-f(x)}{x_1-x}$ is called average rate of change.

2. Derivative: (Sargodha 2009,2017 Fsd 2011, Guj 2013,2017)

Instantaneous rate of change of one variable with respect to other variable is called derivative or if limit of $\lim_{\delta x \to 0} \frac{f(x+\delta x)-f(x)}{\delta x}$ exist then it is called derivative denoted by $\frac{dy}{dx}$.

3. Maclaurin Series:

$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots\ldots$ *Is called* maclaurin series.

4. Taylor Series:

$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \ldots$ is called taylor Series

5. Increasing: (Fsd 2010, Sgd 2014,16)

f is increasing on the interval (a,b) if $f(x_2) > f(x_1)$ where $x_2 > x_1$ for $x_1, x_2 \in (a,b)$

6. Decreasing: (Sargodha 2010,16)

f is decreasing on the interval (a,b) if $f(x_2) < f(x_1)$ where $x_2 > x_1$ for $x_1, x_2 \in (a,b)$

7. Stationary Point: (Guj 2010, Sgd 2015, AJK 2016, Mul 2017, Fsd 2017)

Any point where f is neither increasing nor decreasing.

8. Critical value or Critical Point: (Rwp 2016, Lhr 2017)

If $c \in$ Domain of f and $f'(c) = 0$ or $f'(c)$ does not exist then c is called critical value or critical point.

9. Relative Maxima: (Fsd 2018)

f has relative maxima at c if $f''(c) < 0$

10. Relative Minima:

f has relative minima at c if $f''(c) > 0$

11. Point of Inflection: (Bahwlpur 2016, Lhr 2017)

The function f is increasing before $x = 0$ and also after $x = 0$ such point is called point of inflection.

12. Power series: (Lhr 2018)

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \ldots + a_nx^n + \ldots$ is called power series expansion of a function f(x) where a_0, a_1, a_2 are constant and x is variable

1. $\frac{d}{dx}(c)=0$
2. $\frac{d}{dx}(x)=1$
3. $\frac{d}{dx}(cx)=c.1=c$
4. $\frac{d}{dx}(x^n)=nx^{n-1}$
5. $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
6. $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
7. $\frac{d}{dx}(\ln x)=\frac{1}{x}$
8. $\frac{d}{dx}(e^x)=e^x$
9. $\frac{d}{dx}(a^x)=a^x\ln a$
10. $\frac{d}{dx}(\sin x)=\cos x$
11. $\frac{d}{dx}(\cos x)=-\sin x$
12. $\frac{d}{dx}(\tan x)=\sec^2 x$
13. $\frac{d}{dx}(\cot x)=-\operatorname{cosec}^2 x$
14. $\frac{d}{dx}(\sec x)=\sec x\tan x$
15. $\frac{d}{dx}(\operatorname{cosec} x)=-\operatorname{cosec} x\cot x$
16. $\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$
17. $\frac{d}{dx}(\cos^{-1}x)=\frac{-1}{\sqrt{1-x^2}}$
18. $\frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$
19. $\frac{d}{dx}(\cot^{-1}x)=\frac{-1}{1+x^2}$
20. $\frac{d}{dx}(\sec^{-1}x)=\frac{1}{|x|\sqrt{x^2-1}}$
21. $\frac{d}{dx}(\operatorname{cosec}^{-1}x)=\frac{-1}{|x|\sqrt{x^2-1}}$
22. $\frac{d}{dx}(\sinh x)=\cosh x$
23. $\frac{d}{dx}(\cosh x)=\sinh x$
24. $\frac{d}{dx}(\tanh x)=\operatorname{Sech}^2 x$
25. $\frac{d}{dx}(\tanh^{-1}x)=\frac{1}{1-x^2}$
26. $\frac{d}{dx}(\coth x)=-\operatorname{Cosech}^2 x$
27. $\frac{d}{dx}(\operatorname{Sech} x)=-\operatorname{Sech} x\tanh x$
28. $\frac{d}{dx}(\operatorname{Cosech} x)=-\operatorname{Cosech} x\coth x$
29. $\frac{d}{dx}(\sinh^{-1}x)=\frac{1}{\sqrt{1+x^2}}$
30. $\frac{d}{dx}(\cosh^{-1}x)=\frac{1}{\sqrt{x^2-1}}$
31. $\frac{d}{dx}(\coth^{-1}x)=\frac{1}{1-x^2}$
32. $\frac{d}{dx}(\operatorname{Sech}^{-1}x)=\frac{-1}{x\sqrt{1-x^2}}$
33. $\frac{d}{dx}(\operatorname{Cosech}^{-1}x)=\frac{-1}{x\sqrt{1+x^2}}$

INTEGRATION

3

Definitions

Integration or Antiderivative: Rwp 2016
Inverse process of differentiation is called integration.
$df = f'(x)\,dx$, $f'(x)$ is called **differential co-efficient.**
Fundamental Theorem of Calculus:

If f is continuous on $[a, b]$ and $\varphi'(x) = f(x)$ then $\int_a^b f(x)\,dx = \varphi(b) - \varphi(a)$

Differential Equations: Guj 2018
An equation containing at least one derivation of a dependent variable w.r.t. an independent variable. e.g.

$$y\frac{dy}{dx} + 2x = 0$$

Order of a differential equation: The order of a differential equation is the order of the highest derivative in the equation.
Initial Conditions:
The arbitrary constants involving in the solution of differential equation can be determined by the given condition. Such conditions are called initial value conditions.

Important Formulas

rivative	Comparison	Integration
$\frac{d}{dx}(c) = 0$		1. $\int 0\,dx = c$
$\frac{d}{dx}(x) = 1$		2. $\int 1\,dx = x$
$\frac{d}{dx}(x^n) = nx^{n-1}$		3. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$		4. $\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}(a^x) = a^x \cdot \ln a$		5. $\int a^x\,dx = \frac{a^x}{\ln a} + c$
$\frac{d}{dx}(e^x) = e^x$		6. $\int e^x\,dx = e^x + c$
$\frac{d}{dx}(\sin x) = \cos x$		7. $\int \sin x\,dx = -\cos x$
$\frac{d}{dx}(\cos x) = -\sin x$		8. $\int \cos x\,dx = \sin x$
$\frac{d}{dx}(\tan x) = \sec^2 x$		9. $\int \sec^2 x\,dx = \tan x$

10. $\frac{d}{dx}(\text{Cot}x) = -\text{Cosec}^2x$

11. $\frac{d}{dx}(\text{Sec}x) = \text{Sec}x\,\text{Tan}x$

12. $\frac{d}{dx}(\text{Cosec}x) = -\text{Cosec}x\,\text{Cot}x$

13. $\frac{d}{dx}(\text{Tan}^{-1}x) = \frac{1}{1+x^2}$

14. $\frac{d}{dx}\text{Sin}(nx) = \text{Cos}(nx).n$

15. $\frac{d}{dx}(e^{nx}) = e^{nx}.n$

10. $\int \text{Cosec}^2x\,dx = -\text{Cot}x$

11. $\int \text{Sec}x\,\text{Tan}x\,dx = \text{Sec}x$

12. $\int \text{Cosec}x\,\text{Cot}x\,dx = -\text{Cosec}x$

13. $\int \frac{1}{1+x^2}dx = \text{Tan}^{-1}x+c$

14. $\int \text{Sin}nx\,dx = \frac{-\text{Cos}\,nx}{n}$

15. $\int e^{nx}dx = \frac{e^{nx}}{n}$

16. $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\text{Tan}^{-1}\frac{x}{a}$

17. $\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}$

18. $\int \frac{dx}{\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}$

19. $\int \frac{1}{\sqrt{a^2+x^2}} = ln(x+\sqrt{a^2+x^2})+c$

20. $\int (f(x))^n.f'(x)dx = \frac{(f(x))^{n+1}}{n+1}$

21. $\int \frac{f'(x)dx}{f(x)} = ln|f(x)|$

Introduction to Analytic Geometry

4

Definitions:

Co-ordinate system:
Draw the plane two mutually perpendicular lines intersect at origin divides plane in four equal parts. These lines are called axes and system is called co-ordinate system.

Translation:
Let xy-co-ordinate system be given and O'(h, k) is any point in plane. Through O' draw new perpendicular lines O'x and O'y parallel to Ox and Oy. New axes O'x and O'y are called translation of Ox and Oy.

Slop or Gradient:
The measure of steepness (ratio of rise to run) is termed as slope or gradient denoted by $m = Tan\alpha$

Trapezium:
A quadrilateral is having two parallel and two non-parallel sides.

Homogeneous Equation:
Equation $f(x, y) = 0$ is called homogeneous equation"

Important Formulas

1. Distance = $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (point A (x_1, y_1) to point B (x_2, y_2) (Sgd 2010)
2. Distance = d (from one point to line) = $\dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$
3. Ratio (divide internally) = $\left(\dfrac{k_1x_2 + k_2x_1}{k_1 + k_2}, \dfrac{k_1y_2 + k_2y_1}{k_1 + k_2}\right)$
4. Ration (Divide externally) = $\left(\dfrac{k_1x_2 - k_2x_1}{k_1 - k_2}, \dfrac{k_1y_2 - k_2y_1}{k_1 - k_2}\right)$
5. Mid Point = $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
6. Centroid = $\left(\dfrac{ax_1 + bx_2 + cx_1}{a+b+c}, \dfrac{ay_1 + by_2 + cy_2}{a+b+c}\right)$
7. Equation of Translation $\left.\begin{matrix} X = x - h \\ Y = y - k \end{matrix}\right\}$
8. Equation of Rotation $\left.\begin{matrix} X = xCos\theta + ySin\theta \\ Y = yCos\theta - xSin\theta \end{matrix}\right\}$
9. Slope = $m = Tan\alpha$

10. $m = \frac{y_2 - y_1}{x_2 - x_1}$ (If two points are given)

11. $m = \frac{-a}{b}$ if line $(ax + by + c = 0)$ is given

12. Two lines are parallel if $\boxed{m_1 = m_2}$ *also* $\boxed{a_1 b_2 - a_2 b_1 = 0}$

13. Two lines are perpendicular $\boxed{m_1 . m_2 = -1}$ also $\boxed{a_1 a_2 + b_1 b_2 = 0}$

14. Collinear ; slope of AB = slope of AC

15. Slope intercept form $y = mx + c$

16. Two intercept form $\frac{x}{a} + \frac{y}{b'} = 1$

17. Equation of Line $(y - y_1) = m(x - x_1)$

18. Symmetric form $\frac{x - x_1}{Cos\alpha} = \frac{y - y_1}{Sin\alpha} = r$

19. Normal form $xCos\alpha + ySin\alpha = p$

20. Area of Triangle $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

21. $Tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$

22. $m_1 + m_2 = \frac{-2h}{b}$ & $m_1 m_2 = \frac{a}{b}$

23. $Tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$

24. $h^2 - ab = 0$ then lines are coincident

25. $a + b = 0$ *then* $\theta = 90°$

26. Joint equation $ax^2 + 2hxy + by^2 = 0$

Linear Inequalities and Linear programming

5

Definitions:

Linear Inequalities:

Inequalities are expressed by following symbols $<, >, \leq, \geq$ with one or two variables are called linear inequalities.

Linear programming

(Rwp 2017, Shwl 2017, Fsd 2018)

Linear programming deals with the optimization (maximization or minimization) of the function

Boundary of half plane

$ax + by < c$ is called half plane region and line $ax + by = c$ is called. Boundary of half plane.

Left, Right, Upper, Lower Half Plane:

Vertical line divides the plane into left or right and non-vertical line divides into lower and upper half plane.

Vertex or Corner Point:

(Rwp 2016, Bahl 2016, Fsd 2017, Shwl 2018, Dgk 2018)

A point of a solution region where two of its boundary lines intersect is called vertex.

Non-Negative Constraints:

The variable used in the system of linear inequalities relating to the problem of every day life are non-negative and are called non-negative constraints.

Decision Variables:

(Sargodha 2012)

The non-negative constraints play an important role for taking decision. So these variables are also called Decision Variables.

Solution region:

(Sargodha 2015)

We draw graph of each inequality in the system of the same coordinates axes and then take intersection of the graph. The common region so obtained is called the solution region.

Feasible Region:

(Sgd 2010, 2015,2017, Fsd 2015, Mul 2016, DGK 2016, AJK 2016)

A region which is restricted to the first quadrant is called feasible region.

Feasible Solution:

(Sgd 2014, Fsd 2015, Shw 2016, AJK 2016, Lhr 2018, Mul 2018)

Each point of feasible region is called feasible solution.

11. Optimal Solution: (Fsd 2015, Lhr 2018)

The feasible solution which maximize or minimize the objective function is called the optimal solution.

12. Objective Function: (Fsd 2016, Sgd 2016,2018, Guj 2016, Mul 2016)

A function which is to be maximized or minimized is called an objective function.

13. Problem Constraints: (Fsd 2015)

The system of linear inequalities involved in the problem concerned are called problem constraints.

14. Convex: (Rwp 2015, Lhr 2016, Dgk 2018, Bhwl 2018)

If the line segment obtained by joining any two points of a region lies entirely within the region then the region is called convex.

Conic Sections

6

Definitions

1. Nappes:

Two parts of cone are called nappes.

2. Vertex or Apex:

Meeting point of two parts of cone is called vertex or apex.

3. Circle:

If cone is cut by a plane perpendicular to the axis of cone, then resulting section is circle. (Sargodha 2011)

We can also define circle as:

A locus of a point which remains at a fixed distance from a certain point. The point is called centre of circle and fixed distance is called radius of the circle.

4. Parabola:

If the intersecting plane is parallel to a generator of the cone but cuts one nape only is called parabola. (Sargodha 2008, 18)

5. Ellipse:

If the cone is cut by a plane and the cutting plane is slightly tilted and cuts only one nappe of cone the resulting section is an ellipse.

6. Hyperbola:

If the cone is cut by a plane and the cutting plane is parallel to the axis of cone and intersect both nappes, then curve of intersection is Hyperbola. (Sargodha 2011, Shw 2019)

7. Point circle:

If the plane passes through vertex of cone, the intersection is a single point or point circle or if $r=0$. (Sargodha 2008)

8. Parametric equations:

$x = r\text{Cos}\theta$ $y = r\text{Sin}\theta$ are parametric equations of circle. (Sargodha 2010)

9. Tangent:

A line that touch the curve without cutting through it.

10. Normal:

A line perpendicular to Tangent is called normal.

11. Tangential distance:

Length of tangent is called tangential distance and its formula is $\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

12. Chord of contact:

The line joining points of contact of chord.

13. Conic section:

$\frac{|PF|}{|PM|} = e$ (+ve constant) is called conic section if $e < 1$ then Ellipse, if $e = 1$ then parabola, if $e > 1$ then hyperbola.

Eccentricity:
The number e is called eccentricity.

Equation of parabola
$y^2 = 4ax$ is called equation of parabola, $F(a, 0)$ is focus, A line at equal distance from vertex opposite to focus is directrix. The point where axis meet the parabola is called vertex (0, 0). The line through focus and perpendicular to directrix is called axis of parabola and focal chard perpendicular to axis is called latusrectum.

Vertices
For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$ points A, A' are called vertices and $AA' = 2a$ is called Major axis, B, B' are covertices and $BB' = 2b$ is minor axis.

Transverse or Focal
The line segment $AA' = 2a$ is called Transverse or Focal of Hyperbola and BB' (line segment) is called conjugate ax.

Central Conics:
Ellipse and hyperbola are called central conics.

Define focal chord of parabola (Sgd 2015)
A line joining two distinct point on parabola and passing through Focus is called focal Chord.

Degenerate Conic:
Under certain condition equatic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ not represent as conic. In such case this is called degenerate conic.

Important Formulae

Equation of circle in standard form $(x - h)^2 + (y - k)^2 = r^2$ (Centre (h, k), radius-r) (Sargodha 2011)
If centre is at origin then equation of circle $x^2 + y^2 = r^2$ (Sargodha 2008)
General equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ where centre $= (-g, -f)$ and radius $= \sqrt{g^2 + f^2 - c}$
For circle equation of tangent $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$ at (x_1, y_1)
For circle equation of Normal at
(x_1, y_1) is $(y - y_1)(x_1 + g) = (x - x_1)(y_1 + f)$
Equation of parabola $y^2 = 4ax$ and at (x_1, y_1) is $y^2 = 4ax_1$ whose vertex is at origin.
Standard equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If $a > b$ and $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ If $a > b$ (Sgd 2008)
Standard equation of Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1) $\boxed{\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1}$

$y = mx + \frac{a}{m}$ is tangent to $y^2 = 4ax$
$y = mx \pm \sqrt{a^2m^2 + b^2}$ Tangent of Ellipse
$y = mx \pm \sqrt{a^2m^2 - b^2}$ Tangent of hyperbola.

Vectors

7

Definitions

Scalar A physical quantity which is defined only by its magnitude. For example mass, time, length

Vector A physical quantity defined by its magnitude and direction also. For example force, weight, velocity.

Magnitude or Length or Norm Absolute value of vector is called magnitude or length or Norm $|\vec{AB}|$ Fsd 2018

Unit Vector A vector whose magnitude is unity or 1, $\hat{p} = \frac{\underline{v}}{|\underline{v}|}$ Lhr 2009,2018

Equal Vectors Two vectors $\vec{AB}$ and $\vec{CD}$ are equal if they have same magnitude and direction $|\vec{AB}| = |\vec{CD}|$ Lhr 2009

Parallel Vectors Two vectors are parallel if and only if they are non-zero scalar multiple of each other $\underline{a} = \lambda \underline{b}$

Triangular Law If $\vec{AB}$, $\vec{BC}$ and $\vec{AC}$ are three sides of triangle then $\vec{AB} + \vec{BC} = \vec{AC}$ is called triangular law.

Position Vector The vector of whose initial point is the origin O terminal point is P. (Sgd 2014, Ajk 2009)

Zero Vector If magnitude of a vector is zero then it is called zero vector.

Direction angles and Direction Cosines Let $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ be non-zero vector and α, β, γ are angle formed between $\underline{r}$ and $\underline{i}, \underline{j}, \underline{k}$ respectively then α, β, γ are Direction angles and $\cos\alpha, \cos\beta, \cos\gamma$ are Direction Cosines. (Mirpure 2009, Lhr 2018, Mul 2018)

Scalar or Dot Product If $\underline{u}$ and $\underline{v}$ are non-zero vectors in a plane with same initial line then their dot product is $\underline{u} \cdot \underline{v} = |\underline{u}|\,|\underline{v}| \cos\theta$ (Lhr 2008)

Cross or Vector Product If u & v are non zero vectors then $\underline{u} \times \underline{v} = (|\underline{u}|\,|\underline{v}| \sin\theta)\,\hat{n}$ (Sargodha 2008)

Work done If a constant force F applied to a body act at an angle θ to the direction of motion then work done is work done $= \underline{F} \cdot \underline{d}$ (Shwl 2017)

Important Formulae

1. Vector = $\vec{AB}$ or $\underline{U}$
2. Scalar = AB or U or $|\vec{AB}|$
3. Magnitude = $|\vec{AB}|$
4. Unit Vector = $\hat{p} = \frac{\underline{v}}{|\underline{v}|}$
5. Triangular Law of addition $\vec{AB} + \vec{BC} = \vec{AC}$
6. Equal Vectors $|\vec{AB}| = |\vec{CD}|$
7. Position Vector = $\vec{OP}$
8. Ratio Formula $\underline{r} = \frac{q\underline{a} + p\underline{b}}{p+q}$
9. Direction angles α, β, γ

10. Direction Cosines $\cos\alpha = \frac{x}{r}$, $\cos\beta = \frac{y}{r}$, $\cos\gamma = \frac{z}{r}$

11. Triples can be direction angle if $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

12. Scalar product or Dot product of $\underline{u}$ and $\underline{v}$ $\quad \underline{u}.\underline{v} = |\underline{u}|\ |\underline{v}| \cos\theta$

13. Perpendicular if $\underline{u} \cdot \underline{v} = 0$

14. Parallel if $\underline{u} = \lambda\underline{v}$ $\qquad$ or $\qquad \underline{u} \times \underline{v} = 0$

15. Vector product or Cross Product $= \underline{u} \times \underline{v} = |\underline{u}|\ |\underline{v}| \sin\theta\, \hat{n}$

16. $\underline{u} \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$, where $\underline{u} = u_1\underline{i} + u_2\underline{j} + u_3\underline{k}$, $\underline{v} = v_1\underline{i} + v_2\underline{j} + v_3\underline{k}$

17. Area of parallel gram ABCD $= |\overrightarrow{AB} \times \overrightarrow{AC}|$

18. Area of Triangle ABC $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

19. Volume of parallelepiped $= \underline{u} \cdot \underline{v} \times \underline{w}$

20. $\underline{u}, \underline{v}, \underline{w}$ are coplanar if $\underline{u} \cdot \underline{v} \times \underline{w} = 0$

21. Volume of Tetrahedron $= \frac{1}{6}(\underline{u} \cdot \underline{v} \times \underline{w})$

22. Work done $= \underline{F} \cdot \underline{D}$

23. Moment of Force $= \underline{r} \times \underline{F}$

24. $\underline{u} \cdot \underline{v} \times \underline{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

25. $\underline{u} \cdot \underline{v} \times \underline{w} = \underline{v} \cdot \underline{w} \times \underline{u} = \underline{w} \cdot \underline{u} \times \underline{v}$